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SEPARATUM

ON RELATIONS BETWEEN INFORMATION AND PHYSICS

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(Received December 10, 1983)

Results of a new "gnostical" theory of real data are interpreted in terms of physics. The influence of uncertainty on real data is equivalent to a virtual motion of data along geodesics. Both thermodynamical laws are shown to hold for closed cycles of transformations of data. Changes of information borne by each particular datum evaluated by gnostical theory are shown to satisfy an equation of equivalence between sources of fields of information and of thermodynamical entropy.

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1828

AKADÉMIAI KIADÓ, BUDAPEST
PUBLISHING HOUSE OF THE HUNGARIAN ACADEMY OF SCIENCES
VERLAG DER UNGARISCHEN AKADEMIE DER WISSENSCHAFTEN
MAISON D'EDITIONS DE L'ACADEMIE DES SCIENCES DE HONGRIE
ИЗДАТЕЛЬСТВО АКАДЕМИИ НАУК ВЕНГРИИ

Problems of Control and Information Theory, Vol. 13 (6), pp. 383–399 (1984)

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Results of a new "gnostical" theory of real data are interpreted in terms of physics. The influence of uncertainty on real data is equivalent to a virtual motion of data along geodesics. Both thermodynamical laws are shown to hold for closed cycles of transformations of data. Changes of information borne by each particular datum evaluated by gnostical theory are shown to satisfy an equation of equivalence between sources of fields of information and of thermodynamical entropy.

1. Introduction and summary of previous results

A new approach to the problem of uncertainty of real data has been developed in [1] and [2]. Real data are results of *quantification* which is an empirical procedure, measurements of real quantities and/or counting of real objects. Practical quantification is always affected by uncertainty of various origin. Uncertainty of small data samples is not necessarily governed by a statistical law. But even if it is then a test of statistical hypotheses on a small data sample is problematic because proper statistical statements are related to "sufficiently big" samples. The idea of gnostical theory is not to extrapolate features of small data samples from big ones but to base the theory of small data samples on detailed knowledge of features of each individual datum and on a proper data composition law. Gnostical theory of individual data [1] is based on Axiom 1 stating that each possible datum z may be represented as

$$z = z_0 \xi \quad (z_0, \xi \in R_+) \tag{1}$$

where R_+ is the interval of finite positive real numbers. The quantity z_0 is called the *ideal value*, it is the result of an *ideal quantification*. The quantity ξ characterizes the uncertainty. Using its logarithm Ω we have the model of data

$$z = z_0 e^{\Omega} \tag{2}$$

from which a mathematical model of quantification results:

$$\mathbf{u} = \mathbf{K}_q(\Omega)\mathbf{u}_0 \tag{3}$$

where

$$\mathbf{u} := \begin{pmatrix} z_0 \operatorname{ch} \Omega \\ z_0 \operatorname{sh} \Omega \end{pmatrix} := \begin{pmatrix} x \\ y \end{pmatrix} \quad \mathbf{K}_q(\Omega) := \begin{pmatrix} \operatorname{ch} \Omega & \operatorname{sh} \Omega \\ \operatorname{sh} \Omega & \operatorname{ch} \Omega \end{pmatrix} \quad \mathbf{u}_0 := \begin{pmatrix} z_0 \\ 0 \end{pmatrix}. \tag{4}$$

If z_i is a real datum, the ideal value of which is z_0, then $\Omega_i = \ln(z_i/z_0)$ and the corresponding $\mathbf{u}_i$ determines the "end point" of the quantifying path starting at the point $\mathbf{u}_0$. This path is an arc of a pseudoeuclidean circle which has diameter $z_0 = \sqrt{x^2 - y^2} = \text{const}$. The quantity z_0 is thus an invariant of the quantifying transformation.

Ideal estimation is an optimal transformation of a datum that yields an estimate $\tilde{\mathbf{u}}_0$ coinciding with the ideal value $\mathbf{u}_0$ up to a scale factor. For a real datum z_i parametrized by Ω_i the ideal estimating transformation is

$$\mathbf{u}_i' = \mathbf{K}_e(\omega_i)\mathbf{u}_i = \mathbf{K}_e(\omega_i)\,\mathbf{K}_q(\Omega_i)\mathbf{u}_0 \tag{5}$$

where

$$\mathbf{u}_i' = \begin{pmatrix} r_i \\ 0 \end{pmatrix} \quad \mathbf{r}_i := \sqrt{x_i^2 + y_i^2} \quad \mathbf{K}_e(\omega_i) := \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \tag{6}$$

and where the relation

$$-\operatorname{tg}\omega_i = \operatorname{th}\Omega_i = \frac{y_i}{x_i} \tag{7}$$

holds. Estimating path from $\mathbf{u}_i$ to $\mathbf{u}_i'$ is thus an arc of a Euclidean circle having radius r_i, which is thus the invariant of ideal estimation.

Attenuating transformation is defined as

$$\mathbf{u}_0 = \mathbf{K}_a(k_i)\mathbf{u}_i' \tag{8}$$

where

$$\mathbf{K}_a(k_i) := \begin{pmatrix} k_i & 0 \\ 0 & k_i \end{pmatrix} \qquad k_i := z_0/r_i . \tag{9}$$

For a given datum z_i the parameters Ω_i, ω_i and k_i are fixed numbers, and vectors $\mathbf{u}_0$, $\mathbf{u}_i$ and $\mathbf{u}_i'$ correspond to fixed points of a plane (x, y). For points of paths between them the quantities Ω, ω and k are variables. Considering these free points we find that transformations (3), (5) and (8) constitute the commutative groups G_q, G_e and G_a, respectively. To each $\mathbf{u}^T = (x, y)$ there is a "complement" ${}_c\mathbf{u}^T := (y, x)$. The same transformations as (3), (5) and (8) exist for the complements.

The ideal gnostical cycle IGC (z_0, z_i) (or shortly IGC$_i$) is formed by the quantifying path (from $\mathbf{u}_0$ to $\mathbf{u}_i$), by the estimating path (from $\mathbf{u}_i$ to $\mathbf{u}_i'$) and by the linear attenuating path (from $\mathbf{u}_i'$ back to $\mathbf{u}_0$). It includes not only the defining points $\mathbf{u}_0$, $\mathbf{u}_i$ and $\mathbf{u}_i'$ but also all other points of paths between them.

The varied gnostical cycle VGC (z_0, z_i) (or VGC$_i$) is a closed gnostical cycle passing through the same defining points as IGC$_i$ does ($\mathbf{u}_0 - \mathbf{u}_i - \mathbf{u}_i' - \mathbf{u}_0$). It differs from the IGC$_i$ by small variations of the quantifying path and/or of the estimating path. For complements ${}_c\mathbf{u}$ quite analogous cycles can be defined. The ideal gnostical cycle has been shown in [1] to minimize the unavoidable loss of information due to the contribution of uncertainty.

Quantifying and estimating *changes of information* I_q and I_e, respectively, have been shown in [1] to be given by the expressions

$$I_q = H(1/2) - H(p_q) \qquad I_e = H(1/2) - H(p_e) \tag{10}$$

where

$$H(p) := -p\ln(p) - (1-p)\ln(1-p) \tag{11}$$

$$p_q := (1 + \sqrt{-1}\,h_q)/2 \qquad p_e := (1 + h_e)/2 \tag{12}$$

and where h_q and h_e are elements of matrices $\mathbf{K}_q^2(\Omega)$ and $\mathbf{K}_e^2(\omega)$,

$$\mathbf{K}_q^2(\Omega) = \begin{pmatrix} 1/f & h_q \\ h_q & 1/f \end{pmatrix} \qquad \mathbf{K}_e^2(\omega) = \begin{pmatrix} f & -h_e \\ h_e & f \end{pmatrix}. \tag{13}$$

If $\operatorname{tg}\omega = -\operatorname{th}\Omega$ then *the quantifying irrelevance* h_q, *the estimating irrelevance* h_e and *the fidelity* f are mutually related by the identities

$$f = \frac{x^2 - y^2}{x^2 + y^2} = \frac{2}{\xi^2 + \xi^{-2}} = \frac{1}{\operatorname{ch} 2\Omega} = \cos 2\omega \tag{14}$$

$$h_q = \frac{2xy}{x^2 - y^2} = \frac{\xi^2 - \xi^{-2}}{2} = \operatorname{sh} 2\Omega = -\frac{h_e}{f} \tag{15}$$

$$h_e = \frac{-2xy}{x^2 + y^2} = -\frac{\xi^2 - \xi^{-2}}{\xi^2 + \xi^{-2}} = -\operatorname{th} 2\Omega = \sin 2\omega = -h_q f \tag{16}$$

where

$$\xi := z/z_0 . \tag{17}$$

A data sample denoted $Z(z_0, n)$ or shortly Z is an n-tuple of real data $z_1, \ldots, z_n$ the ideal value of which is z_0. A function of all $z_i \in Z$ and of z_0 is a *characteristic* of the data sample Z. Axiom 2 of the gnostical theory [2] is the composition rule for the evaluation of characteristics of data samples: Let $Z(z_0, n)$ be a data sample. Let $\mathbf{u}_{cq}^T := (z_0 \operatorname{ch}\Omega_c, z_0 \operatorname{sh}\Omega_c)$ and $\mathbf{u}_{ce}^T = (r_c \cos\omega_c, -r_c \sin\omega_c)$ be composite vectors of the data sample Z. Let Ω_c, ω_c and r_c be characteristics of Z. Then Axiom 2 states:

$$\mathbf{K}_q^2(\Omega_c) = \frac{1}{w_q^2}\sum_i^n \mathbf{K}_q^2(\Omega_i) \qquad \mathbf{K}_e^2(\omega_c) = \frac{1}{w_e^2}\sum_i^n \mathbf{K}_e^2(\omega_i) \tag{18}$$

where

$$w_q^2 = \operatorname{Det}\left\{\sum_i^n \mathbf{K}_q^2(\Omega_i)\right\} \qquad w_e^2 = \operatorname{Det}\left\{\sum_i^n \mathbf{K}_e^2(\omega_i)\right\}. \tag{19}$$

As shown in [2] the characteristics resulting from this composition rule approach the statistical characteristics if and only if all relative values $\xi_i = z_i/z_0$ ($z_i \in Z$, $\forall_i$) are close to 1 (the case of weak uncertainties). If uncertainties are not weak then the gnostical characteristics differ substantially from the statistical ones in that they are less sensitive to outlying or inlying data. Formulae for *practical estimations* having such desirable features and using only data have been derived from Axiom 2 in [2]. They are optimal in that they approach in a sense the ideal gnostical cycle as close as possible.

The aim of this paper is to propose interpretations of both axioms mentioned and of the results derived formally from them and to show that deep interrelations exist between information and other gnostical characteristics on one hand and fundamental notions of physics such as entropy, energy, time and space on the other hand.

2. Correspondence between Axiom 1 and the measurement theory and practice

Quantification is an important part of all cognitive processes as a necessary condition of achieving the highest level of knowledge of reality. It includes both measurement and counting of real quantities. Theory of measurement initiated by v. Helmholtz (1887) characterizes the measurement as a mapping of empirical relational structures and their relations and operations onto the mathematical ones. The specific of this mapping is that it is a bridge between reality and mathematics. Empirical objects are always much more complicated than the mathematical ones. Empirical notion of the relation "to be equal" between empirical objects has been developing for thousands years together with measuring technology and instruments. The technology of measurement includes necessarily a measuring unit. Results of measurements say how many times the measured quantity is greater or smaller than the unit. Therefore, real data obtained by measurements are not arbitrary numbers, they are positive and finite. Neither zero nor infinity can be taken as a result of a proper measurement. The same holds for results of counting. Axiom 1 (1) of gnostical theory expresses thus the basic fact of both theory and practice of the quantification. There are of course many reasons to use data which have been transformed: to use shifted zero and bipolar data. But to treat such data, it is necessary to transform them back to the proper fundamental form before substitution into gnostical formulae. Therefore, the notion "real data" within the presented theory does not include just any data originating from reality but only data having the fundamental form. This is important because only then can Axiom 1 serve its purpose: to accommodate real data in a theory.

Theory of measurement does not include effects of uncertainty. In Axiom 1 (1) such effects are represented by the factor ξ. But the linearity of results of quantification with respect to the ideal value $\mathbf{z}_0$ corresponds to another important assumption of measurement theory: the mapping of quantities onto numbers is taken to be linear.

Axiom 1 of the gnostical theory with its important consequences is thus based on long-time experience of quantitative cognition of the reality.

3. Gnostical tensors

Under "*gnostical*" we shall understand "quantifying" and/or "estimating". Gnostical *event* will be any two-dimensional description of a result of practical quantification or of its transform, e.g. $\mathbf{u}$ having either the quantifying or the estimating form:

$$\mathbf{u} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} z_0 \operatorname{ch} \Omega \\ z_0 \operatorname{sh} \Omega \end{pmatrix} = \begin{pmatrix} r \cos \omega \\ -r \sin \omega \end{pmatrix} \tag{20}$$

where

$$r = \sqrt{x^2 + y^2}\,. \tag{21}$$

Both quantifying and estimating transformations of events of a general type $\mathbf{u}' = \mathbf{K}_q(\Omega)\mathbf{u}$ and $\mathbf{u}' = \mathbf{K}_e(\omega)\mathbf{u}$ can be unified: Let $\mathbf{v}^T := (x, by)$ where $b = 1$ in the case of quantification and $b = \sqrt{-1}$ in the case of estimation. The event $\mathbf{v}$ may be interpreted as a two-vector v^α ($\alpha = 1, 2$). Let K_α^β be an element $(K)_{\alpha,\beta}$ of the matrix

$$\mathbf{K} := \begin{pmatrix} C & bS \\ bS & C \end{pmatrix} \tag{22}$$

where $C = \operatorname{ch} \Omega, S = \operatorname{sh} \Omega, b = 1$ for quantification and $C = \cos \omega, S = \sin \omega, b = \sqrt{-1}$ for estimation. Then both versions of (20) may be written in the tensor notation as

$$v'^\beta = K_\alpha^\beta v^\alpha \qquad (\alpha, \beta = 1, 2)\,. \tag{23}$$

Using the same b we obtain the invariants of the transformation (23) for both cases in the form $x^2 - b^2 y^2$.

Theorem 1. Let gnostical transformation (23) take place. Then the matrix $\mathbf{K}^2$ transforms as a twice contravariant tensor.

Proof. The direct product of the two-vector v'^β (23) with the same vector written as $v'^\gamma = K_\delta^\gamma v^\delta$ multiplied by an invariant $a = \dfrac{2}{x^2 - b^2 y^2}$ gives

$$a v'^\beta v'^\gamma = K_\alpha^\beta K_\delta^\gamma a v^\alpha v^\delta\,.$$

Such a transformation does not change the Minkowski tensor $g^{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$:

$$g'^{\beta\gamma} = K_\alpha^\beta K_\delta^\gamma g^{\alpha\delta} \quad \text{and} \quad g'^{\alpha\beta} = g^{\alpha\beta}\,.$$

Therefore

$$Q'^{\beta\gamma} = \frac{\partial v'^\beta}{\partial v^\alpha} \frac{\partial v'^\gamma}{\partial v^\delta} Q^{\alpha\delta}$$

where

$$Q^{\alpha\delta} = a v^\alpha v^\delta - g^{\alpha\delta} \quad \text{and} \quad K_\alpha^\beta = \frac{\partial v'^\beta}{\partial v^\alpha}$$

proves that $Q^{\alpha\delta}$ is a tensor. Its components equal to that of $\mathbf{K}^2$. ∎

Corollary 1.1. Both the quantifying and estimating versions of the composition rule (18) (Axiom 2) correspond to addition of tensors $\mathbf{K}^2$. ∎

Proof. Denoting

$$\mathbf{B} := \begin{pmatrix} 1 & 0 \\ 0 & b \end{pmatrix} \tag{24}$$

we obtain for $b=1$ and $b=\sqrt{-1}$ equalities

$$\mathbf{K}_q^2(\Omega_i) \equiv (\mathbf{K}^2(\Omega_i))_{b=1} \qquad \mathbf{K}_e^2(\omega_i) = (\mathbf{B}^{-1}\mathbf{K}^2(\omega_i)\mathbf{B})_{b=\sqrt{-1}}\,,$$

respectively. Therefore

$$\sum_i \mathbf{K}_q^2(\Omega_i) = \left(\sum_i \mathbf{K}^2(\Omega_i)\right)_{b=1} \qquad \sum_i \mathbf{K}_e^2(\omega_i) = \left(\mathbf{B}^{-1}\left(\sum_i \mathbf{K}^2(\omega_i)\right)\mathbf{B}\right)_{b=\sqrt{-1}}\,.$$ ∎

Corollary 1.2. Inverse fidelity f^{-1} (14) and quantifying irrelevance h_q (15) as well as fidelity f (14) and estimating irrelevance h_e (16) are components of a gnostical tensor $\mathbf{K}^2$ corresponding to a matrix $\mathbf{K}_q^2(\Omega)$ or $\mathbf{K}_e^2(\omega)$, respectively. ∎

The importance of tensor properties of squared gnostical operators lies in their connections with some physical quantities.

4. Correspondence between gnostical theory of data and relativistic mechanics

Gnostical theory considers real data of a quite arbitrary nature. It is however natural to require suitability of such a theory also for a very special case of data describing simple events of relativistic mechanics. Let us show that such a requirement is satisfied by the gnostical theory.

A relativistic event is a four-vector x^α ($\alpha = 1, 2, 3, 0$) where the component x^0 is ct (observed time coordinate multiplied by speed of light c), the other components being space coordinates. A Euclidean orthogonal rotation of the coordinate system (x^1, x^2, x^3) does not change observed lengths, time intervals, energies and masses depending on the velocity of the coordinate system. Therefore, from this point of view it is sufficient to consider only a two-vector $\mathbf{w}^T := (ct, s)$ where

$$s = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}\,. \tag{25}$$

Let us consider the simplest relativistic event: A lamp placed at the point $s=0$ flashed for τ seconds. Let us denote $\mathbf{w}_0^T := (c\tau, 0)$. An observer moving with respect to the lamp with velocity v will observe an event $\mathbf{w}$,

$$\mathbf{w} = \mathbf{L}\mathbf{w}_0 \tag{26}$$

where

$$\mathbf{L} = \begin{pmatrix} \gamma & \gamma v/c \\ \gamma v/c & \gamma \end{pmatrix} \qquad (\gamma = (1 - v^2/c^2)^{-1}) \tag{27}$$

is the operator of Lorentz's transformations. The invariant of this transformations is the proper interval $c\tau = \sqrt{c^2t^2 - s^2}$. There exists a dual relativistic event: Let $s_0 = c\tau$ be the length of a stick observed in the coordinate system fixed at the stick. Such an event is ${}_c\mathbf{w}^T := (0, s_0)$. It will be observed as ${}_c\mathbf{w}^T := (s, ct)$ by an observer moving with respect to the stick with velocity v and the equality

$${}_c\mathbf{w} = \mathbf{L}\,{}_c\mathbf{w}_0 \tag{28}$$

will hold with the same $\mathbf{L}$ (27). The invariant will be the same $s_0 = c\tau$ independently of the velocity v. The relative velocity of the observer's coordinate system is in both cases

$$v/c = s/ct\,. \tag{29}$$

If both s and t are observed then there is no uncertainty in such relativistic observations and the proper quantities $c\tau$ or s_0 as well as the velocity v can be calculated. But a different situation occurs when only the sum $ct + s$ is available for observations. Such an observation may be called *incomplete observation*. In such a case we identify ct with x and s with y. The relativistic event $\mathbf{w}$ is thus identified with a gnostical event u (4) in this case and ${}_c\mathbf{w}$ with the gnostical event ${}_c\mathbf{u}^T = (y, x)$. Then the uncertainty of incomplete relativistic observation will be characterized quantitatively by a real parameter Ω for which the identity

$$\operatorname{th}\Omega = y/x \equiv s/ct \tag{30}$$

holds. We obtain an identity

$$\mathbf{L} = \mathbf{K}_q(\Omega) \tag{31}$$

and the correspondence of the Lorentz's transformations (26) and (28) with quantifying transformations (3). Incompletely observed relativistic events can be thus interpreted as some gnostical events and vice versa. The quantifying uncertainty is then modelled by the unknown velocity of the coordinate system and the quantifying transformations by Lorentz's transformations and vice versa.

There is an important correspondence between the quantifying tensor (represented by the matrix $\mathbf{K}_q^2(\Omega)$) and the energy-momentum tensor of relativistic mechanics. Using the notation introduced above we represent a relativistic event in the form ($c\tau \operatorname{ch}\Omega$, $c\tau \operatorname{sh}\Omega$) where τ is Lorentz's invariant proper time. The two-vector of the velocity is therefore ($c \operatorname{ch}\Omega$, $c \operatorname{sh}\Omega$) and the energy-momentum tensor $T^{\alpha\beta}$ ($\alpha, \beta = 1, 0$) of a flow of relativistic particles having the local density of masses μ has the following components: $T^{10} = \mu c^2 \operatorname{ch}^2\Omega$, $T^{01} = T^{10} = \mu c^2 \operatorname{sh}\Omega \operatorname{ch}\Omega$, $T^{00} = \mu c^2 \operatorname{ch}^2\Omega$.

To distinguish between mathematical statements and statements using facts taken from outside mathematics, the latter will be stated below as Propositions, not Theorems.

Proposition 1. Let $(c\tau \operatorname{ch} \Omega, c\tau \operatorname{sh} \Omega)$ be a relativistic event giving rise to an energy-momentum tensor $T^{\alpha\beta}$. Let $(z_0 \operatorname{ch} \Omega, z_0 \operatorname{sh} \Omega)$ be the gnostical event corresponding to this event and let $\mathbf{K}_q(\Omega)$ be its quantifying operator.

Then there exists a linear dependence of the quantifying tensor represented by $\mathbf{K}_q^2(\Omega)$ on the tensor $T^{\alpha\beta}$.

Proof. Writing the tensor $T^{\alpha\beta}$ in the matrix representation

$$\mathrm{T} = \mu c^2 \begin{pmatrix} \operatorname{ch}^2 \Omega & \operatorname{sh} \Omega \operatorname{ch} \Omega \\ \operatorname{sh} \Omega \operatorname{ch} \Omega & \operatorname{sh}^2 \Omega \end{pmatrix} \tag{32}$$

and using the Minkowskian matrix

$$\mathrm{g}_M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{33}$$

we get

$$\mathbf{K}_q^2(\Omega) = \frac{2}{\mu c^2} \mathrm{T} - \mathrm{g}_M . \tag{34}$$

∎

The importance of this statement consists in that it supports Axiom 2 (18) of gnostical theory: Energy-momentum tensors corresponding to individual particles are added in physics to obtain an energy-momentum tensor of the whole system. In this way the conservation laws are respected. Because of linearity of Eq. (34) the quantifying tensors of gnostical events corresponding to individual particles are to be added, too, to preserve the correspondence between the tensor of the system of particles and the tensor of the composite gnostical event.

Nevertheless, it cannot be concluded that gnostical theory of quantification is nothing but an application of relativistic mechanics. The gnostical theory has been built up on Axiom 1 which is quite different from relativistic axioms and has a much broader field of application. The point of these considerations is to show that a natural requirement for a general model is here satisfied: it is applicable to a very special, maybe even trivial case. It would not be easy to demonstrate such a correspondence for a statistical model of the uncertainty.

5. Gnostical kinematics of real data

Let us consider a subclass of gnostical events for which the coordinate x represents a dimensionless change of observed time while y is a dimensionless change of a time-dependent coordinate of a real object, but not necessarily its geometrical

position. This class is thus much broader than the class of events of relativistic mechanics. The ideal value z_0 to be quantified is either an interval of the proper time (gnostical event $\mathbf{u}$ "of the first kind") or a proper change of the space coordinate (the event $_s\mathbf{u}$ "of the second kind"). The component $x = z_0 \operatorname{ch} \Omega$ differs from z_0 because of the uncertainty which causes also that $y = z_0 \operatorname{sh} \Omega$ differs from zero. x is more relevant and y less relevant with respect to ideal value z_0 and their ratio $y/x = \operatorname{th} \Omega$ is entirely determined by the uncertainty, it characterizes the irrelevance (15), (16) and does not depend on z_0. For a fixed uncertainty (Ω = constant) both x and y increase or decrease proportionally to the ideal value z_0 which in turn increases or decreases with the real quantity. The velocity of variations of the real quantity is, however, not limited by the speed of light in the general case being considered here. For example, a sudden devaluation of currency causes a real motion of economical data without changing any physical quantities the velocity of which is limited. This is an example of a real motion of a gnostical event of the second kind observed under a fixed relative uncertainty. Generalized real motion can be modelled as attenuation or amplification and characterized by a logarithmic variable

$$\kappa = \ln \frac{z_0}{z_{00}} = \ln \frac{c\tau}{z_{00}} \tag{35}$$

where z_{00} is a constant. But both quantities 2Ω and 2ω have a similar structure

$$2\Omega = 2\operatorname{arcth}\left(\frac{y}{x}\right) = \ln \frac{x+y}{x-y} \tag{36}$$

$$2\omega = -2\operatorname{arctg}\left(\frac{y}{x}\right) = -\frac{1}{i} \ln \frac{x+iy}{x-iy} \qquad (i = \sqrt{-1}) . \tag{37}$$

As already mentioned, the quantity τ may be the proper time of a real event in the case of an event of the first kind which is a generalization of a "time-like" relativistic event. But even in the case of an event of the second kind (see example above) the quantity $z_0 = c\tau$ remains to be a parameter of such an event, the independent variable. (Such an event is a generalization of a "space-like" relativistic event.) We can therefore consider the quantity τ to be a generalization of the proper real time. It is not strange that also a "space" variable will be measured by a time interval. Distances between stars can be measured by time intervals as well as economical quantities can ("time is money").

A change of a generalized "space" variable versus generalized "time" variable is a motion. We are thus coming to a *real motion of a general kind* measured by the logarithmic time variable τ. The generalized time variable τ can change in the case of events of the second kind in both directions (because e.g. prices may — at least theoretically — fall as well as rise). The quantity r (6) is a Euclidean analog of this generalized real time.

Using this generalization of the real motion and the analogy demonstrated by (35)–(37) we may interpret the quantification and estimation as two kinds of *gnostical motions*.

The pseudoeuclidean angle 2Ω and Euclidean angle 2ω play role of the *gnostical time*. The three kinds of motions are modelled by three groups of transformations G_q, G_e and G_a. The velocity of each motion of an event is equal to its time derivative:

Table 1. Summary of three kinds of motions of events $\mathbf{u}^T=(x, y)$, ${}_e\mathbf{u}^T=(y, x)$, ${}_\varepsilon\mathbf{u}^T=(-y, x)$

Motion	Group	Logarithmic time	Velocity	Invariant	Variable
Generalized real	G_a	κ	$\frac{d\mathbf{u}}{dz_0}, \frac{d_e\mathbf{u}}{dz_0}, \frac{d\mathbf{u}}{dr}, \frac{d_\varepsilon\mathbf{u}}{dr}$	Ω, ω	z_0, r
Gnostical quantifying	G_q	Ω	$\frac{d\mathbf{u}}{d\Omega}, \frac{d_e\mathbf{u}}{d\Omega}$	z_0	ω, r
Gnostical estimating	G_e	ε	$\frac{d\mathbf{u}}{d\omega}, \frac{d_\varepsilon\mathbf{u}}{d\omega}$	r	Ω, z_0

It can be easily proved that the paths of generalized real motion and of the quantifying motion are orthogonal in pseudoeuclidean metric. Analogically, the paths of generalized real motion and the estimating motion are orthogonal in Euclidean metric.

Thus, we may speak of kinematics of a generalized real motion and kinematics of both quantifying and estimating gnostical motions. Forces driving the generalized real motion lie outside the gnostical theory. The quantifying motion is caused by uncertainty of quantification while the estimating gnostical motion is a result of an effort of a subject to compensate the effect of uncertainty on data as much as possible.

Gnostical motion of both kinds is governed by certain variational principles as derived in [1] from Axiom 1, it is thus a motion along geodesics. The theory of gnostical motion may be therefore interpreted as kinematics of real data.

6. Gnostical dynamics of real data

6.1. Energy of a real datum

Dynamics is dealing with forces but forces are closely connected with variations of energy. Gnostical dynamics includes consideration of energies as a basis for consideration of the entropy of real data.

If a quantity q is measured in terms of electrical current, voltage, or in terms of electrical pulses, then the electrical energy connected with this quantity is proportional to q^2. It may be measured without electrical transformations as well but the measuring process or the dependence of q on time or another quantity may be modelled using an analog computer. Then the energy of the electrical quantity modelling the q is again proportional to q^2. The coefficient of proportionality is a matter of scale, we may therefore take q^2 to be the *energy* of the quantity q.

For a vector quantity $\mathbf{q}$ the energy may be attached to the square $\|\mathbf{q}\|^2=\mathbf{q}^T\mathbf{g}\mathbf{q}$ of the length (where $\mathbf{g}$ is a weighting matrix which corresponds to the metric tensor). This is natural e.g. for a vector $\mathbf{q}$ modelled by velocity of a particle, in which case the energy $\|\mathbf{q}\|^2$ is proportional to the kinetic energy. To obtain positive kinetic energy for both kinds of vector events q in the case of Minkowskian metric we shall use the energy $q_M^2=|\mathbf{q}^T\mathbf{g}_M\mathbf{q}|$ where $\mathbf{g}_M$ denotes the Minkowskian matrix (33).

In considerations of gnostical kinematics of data we have met two velocities of gnostical motion, $\frac{d\mathbf{u}}{d\Omega}\left(\text{or } \frac{d_e\mathbf{u}}{d\Omega}\right)$ and $\frac{d\mathbf{u}}{d\omega}$. These vectors have the same length when the same metric is applied because the equalities

$$\frac{d\mathbf{u}}{d\Omega}=\begin{pmatrix} y \\ x \end{pmatrix} \qquad \frac{d\mathbf{u}}{d\omega}=\begin{pmatrix} -y \\ -x \end{pmatrix} \tag{38}$$

hold. Thus using Minkowskian and Euclidean metric we obtain two energetical variables characterizing gnostical event $\mathbf{u}$:

$$E_-=x^2-y^2 \qquad E_+=x^2+y^2, \tag{39}$$

respectively. The event $\mathbf{u}$ corresponds to a datum $z=x+y$. Let z_i be a fixed real datum obtained as a result of quantification of an ideal value z_0. Let the event $\mathbf{u}$ follows the path of the ideal gnostical cycle $(\text{IGC})_i$. Then both energies E_- and E_+ and their difference E_+-E_- evolve as shown in Table 2.

It is clear from (39) that the increase of the energy E_+ during quantification is due to the contribution of uncertainty because $E_+-E_-=2y^2$. This increase of energy is fully compensated during estimation. Therefore, the following statement results from Table 2.

Table 2. Evolution of energies E_- and E_+ of a gnostical event and of their difference during the ideal gnostical cycle

Phase of the ideal gnostical cycle	Energy		Change of the energy
	E_-	E_+	$E_+ - E_-$
Quantification	invariant ($E_- \equiv z_0^2$)	variable (from z_0^2 to r_i^2)	variable (from 0 to $2y^2$)
Estimation	variable (from z_0^2 to r_i^2)	invariant ($E_+ \equiv r_i^2$)	variable (from $2y_i^2$ to 0)
Attenuation	variable (from r_i^2 to z_0^2)	variable (from r_i^2 to z_0^2)	invariant ($E_+ - E_- = 0$)

Proposition 2. Total change of energy of a gnostical event within an ideal gnostical cycle equals to zero. ▮

This is a specific form of the energy conservation law as manifested by the ideal gnostical cycle.

6.2. *Gnostical channels and their temperature*

Both quantificating and estimating transformations have been unified into the form (23). This linear transformation can be interpreted as a linear *channel* with input v^α and output v'^β. Both versions of this channel have their lengths measured by the parameters Ω or ω. They both have their invariants equal to E_- or E_+. This means that the event v^α keeps its input energy while passing through the channel.

There exists a close connection between energy, amount of heat and temperature. Energy of different kinds can be converted into its thermal form and heat can be measured by temperature. Let us consider the simplest case of linear dependence of absolute temperature on the energy. Then we can speak of *the temperature of a gnostical channel* which is proportional to the invariant input energy of the event v^α. We are thus coming to the temperature of the quantifying channel T_q and of the estimating channel T_e:

$$T_q = c_t E_- \qquad T_e = c_t E_+ \tag{40}$$

where c_t is a constant.

6.3. *Entropy of a real datum*

The thermodynamical entropy is defined by the integral

$$S = \int \frac{dQ}{T} \tag{41}$$

where Q is an amount of heat and T the absolute temperature at which the heat dQ is transferred into or from the system.

Proposition 3. Let z_i be a real datum, the result of quantification of an ideal value z_0. Let f_i be the fidelity (14) of this datum. Then changes $S_{qi} - S_{q0}$ and $S_{ei} - S_{ei0}$ of thermodynamical entropy due to the quantifying and estimating phase of the ideal gnostical cycle $IGC(z_0, z_i)$ are

$$S_{qi} - S_{q0} = k(f_i^{-1} - 1) \qquad S_{ei} - S_{ei0} = k(f_i - 1) \tag{42}$$

where c is a constant. ▮

Proof. For both quantification and estimation we have from Table 2

$$dQ = c_h d(E_+ - E_-) \tag{43}$$

where c_h is a constant. Temperature T_q and T_e (40) are invariants of quantification and estimation, respectively. Therefore

$$S_{qi} - S_{q0} = \frac{c_h}{c_t}(x_i^2 - y_i^2)^{-1} \int_0^{y_i^2} 2dy^2 = \frac{c_h}{c_t}\left(\frac{x_i^2 - y_i^2}{x_i^2 + y_i^2} - 1\right) = \frac{c_h}{c_t}(f_i^{-1} - 1). \tag{44}$$

The quantity $S_{ei} - S_{ei0}$ can be obtained quite analogously. ▮

6.4. *The second law of thermodynamics for a closed cycle of transformations of a real datum*

Let us further consider a real datum z_i the ideal value of which would be z_0.

MAIN PROPOSITION A. Let $IGC(z_0, z_i)$ and $VGC(z_0, z_i)$ be the ideal and varied gnostical cycles respectively. Let $z_i \neq z_0$. Let dS denote successively dS_q, dS_e and dS_a, the changes of entropy S during the quantifying, estimating and attenuating phases of the cycles, respectively. Then

$$\oint_{VGC_i} dS \geq \oint_{IGC_i} dS > 0. \tag{45}$$

▮

Proof. Entropy does not change during attenuation because during this phase equality $E_+ = E_-$ holds. Therefore by (42)

$$\oint_{IGC_i} dS = S_{qi} - S_{q0} + S_{ei} - S_{ei0} = c(f_i^{-1} + f_i - 2) > 0 \tag{46}$$

where $f_i < 1$ because $z_i \neq z_0$.

The sum of changes of entropy in (47) may be rewritten using (14) as

$$S_{qi}-S_{q0}+S_{ei}-S_{ei0}=c(1/\mathrm{ch}\,2\Omega_i+\cos 2\omega_i-2). \tag{47}$$

It has been proved in [1] that the quantity $|\Omega_i|$ is the relative length of the quantifying path of the IGC_i and that varied paths have a shorter relative length. Analogously, the quantity $|\omega_i|$ is the relative length of the estimating path of the IGC_i and it is the minimum of lengths of the varied paths. Therefore the left inequality in (45) follows. ▮

The ideal gnostical cycle is thus optimal not only from the point of view of losses of information (as shown in [1]), it is also thermodynamically optimal in the sense of the left inequality in (45). But even in the case of an ideal gnostical cycle the entropy increases after a gnostical event (corresponding to the datum z_i which does not equal the z_0) passes through the closed cycle.

6.5. A thermodynamical interpretation of gnostical tensors

Both S_q and S_e are functions of the fidelity f (42) which is defined on the variety of events $\mathbf{u}=(x, y)$ and ${}_c\mathbf{u}^T=(y, x)$. This variety has two metrics: Minkowskian and Euclidean ones. Therefore, we can consider the quantities $S_q(x, y)$ and $S_e(x, y)$ to be scalar fields over Minkowski plane M_2 and Euclidean plane E_2, respectively.

Proposition 4. The contravariant gradient of the field $S_q(x, y)$ over the plane M_2 is a two-vector

$$\nabla^{\alpha}_{M}\{S_q\}=-c^{-1}\frac{2h_q}{z_0^2}\frac{d\mathbf{u}}{d\Omega} \tag{48}$$

and the gradient of the field $S_e(x, y)$ over the plane E_2 is

$$\nabla_E\{S_e\}=-c^{-1}\frac{2h_e}{r^2}\frac{d\mathbf{u}}{d\omega}. \tag{49}$$

▮

Proof. The covariant form of the gradient of a scalar field $S_q(x, y)$ is the two-vector

$$\nabla_{\alpha}\{S_q\}=\left(\frac{\partial S_q}{\partial x}, \frac{\partial S_q}{\partial y}\right) \tag{50}$$

the covariant form of which can be obtained by rising the index α. This is effected by multiplying (50) by the metrical tensor:

$$\nabla^{\alpha}_{M}\{S_q\}=\left(\frac{\partial S_q}{\partial x}, -\frac{\partial S_q}{\partial y}\right)=-c^{-1}\left(\frac{4xy^2}{(x^2-y^2)^2}, \frac{4x^2y}{(x^2-y^2)^2}\right). \tag{51}$$

Using expressions (15), (20) and (38) we come to (48). The Euclidean case is analogical, only the difference between covariant and contravariant forms of the gradient disappears. ▮

Corollary 4.1. Let $\|\nabla^{\alpha}_{M}\{S_q\}\|_M$ and $\|\nabla_E\{S_e\}\|_E$ denote lengths of both gradients of entropy measured in corresponding metrics. Then

$$h_q=\frac{z_0}{2\sqrt{-1}}c^{-1}\|\nabla^{\alpha}_{M}\{S_q\}\|_M \qquad h_e=\frac{r}{2}c^{-1}\|\nabla_E\{S_e\}\|_E. \tag{52}$$

▮

Both irrelevances may be thus interpreted as intensities of the diffusion flows of thermodynamical entropy in planes M_2 and E_2. Because both inverse fidelity f^{-1} and fidelity f are linear functions of the entropy (see (42)) we may interpret both versions of the gnostical tensor $\mathbf{K}^2$ (Theorem 1) as tensors of entropy-entropy flow. We have already seen that the tensor $\mathbf{K}_q^2$ corresponds to the energy-momentum tensor of relativistic mechanics (Proposition 1). We now see that the role of relativistic energy is played by entropy in both gnostical cases (quantification and estimation). However, both gnostical tensors have been obtained originally as geometrical "dissimilarity" tensors for a very general case. Their application field is therefore much broader than that of relativistic physics.

6.6. A law of equivalence of the thermodynamical entropy and of the information of a gnostical event

The correspondence between information and thermodynamics has been an object of scientific interest for decades. The gnostical theory offers a mathematical model of such correspondence relating to each particular real datum.

MAIN PROPOSITION B. Let S_q and S_e be thermodynamical entropies of a gnostical event during quantification and estimation, respectively. Let $\nabla^2_M\{S_q\}$ and $\nabla^2_E\{S_e\}$ denote the Laplace's operators $\nabla^2\{\cdot\}=\underset{x,y}{\mathrm{divgrad}}\{\cdot\}$ in Minkowskian and Euclidean metric, respectively. Let $I_q(h_q)$ and $I_e(h_e)$ be quantifying and estimating changes of information, respectively, defined by (10)–(13) and (15), (16). Then it holds

$$(\nabla^2_M\{S_q(x, y)\})_{x',y'}=c_e\left(\frac{d^2I_e}{dh_e^2}\right)_{x',y'} \tag{53}$$

where $x'^2+y'^2=r'^2$ and where both r'^2 and c_e are constants.

$$(\nabla^2_E\{S_e(x, y)\})_{x'',y''}=c_q\left(\frac{d^2I_q}{dh_q^2}\right)_{x'',y''} \tag{54}$$

where $x''^2-y''^2=z_0^2$ and where both z_0^2 and c_q are constants. ▮

Proof. Apply the operator div $\{\cdot\}$ to gradients (48) and (49), differentiate the quantities I_e and I_q and use the invariancy conditions for r'^2 and z_0^2. ∎

The left-hand sides of (55) and (56) are proportional to the intensities of sources of fields of entropy over M_2 and E_2, respectively. The right-hand sides are proportional to intensities of the information on the intervals of both irrelevancies. The correspondence of entropy and information take thus place not at the level of these quantities (fields) but at the level of sources of fields.

The explicit form of the functional relationship between information and entropy may be easily obtained from (53), (54), (42) and (14). Both entropy and information can be evaluated numerically for each data sample as functions of the argument z_0.

7. Relativistic cybernetics, relativistic statistics?

The notion "relativistic cybernetics" appeared probably for the first time in a paper of G. Jumarie who has been publishing a series of papers on this subject for a long time (see Jumarie (1975) and his book [4]). Jumarie's idea can be perhaps formulated as a necessity to take into account the subjectivity of the observer in all cybernetical considerations. G. Jumarie supported his point of view by theoretical arguments. They differ from that of the gnostical theory and the level of the considerations is very general. But the correspondence between relativistic and gnostical events demonstrated above supports the point of view of G. Jumarie in a quite particular and quantitative way. A question may be raised on the necessity and usefulnes of such "complications". Cybernetics uses statistics. Relativistic statistics already exists for a long time de facto, although it is hidden under the name "robust statistical theory". In the framework of the robust statistics, individual random events are given individual weights depending on the uncertainty of the events. This is an analogy of the dependence of masses of particles on the relative velocity of the observers. It has been shown above that using the gnostical model of uncertainty we may obtain a consequent correspondence of this type. It has been shown in [2] how practically applicable estimating algorithms can be derived from the gnostical theory, making it possible to handle small samples of poor data. If this is true then this is a good reason to consider both theoretical and practical problems of relativistic cybernetics.

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О взаимосвязи между информацией и физикой

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В статье интерпретируются результаты новой, так называемой «гностической», теории действительных данных с применением понятий физики. Влияние неопределенностей на действительные данные эквивалентно виртуальному движению данных вдоль геодезических линий. Доказывается, что для замкнутых циклов преобразований данных справедливы оба термодинамических закона. Изменения информации, содержащейся в каждом отдельном элементе набора данных, оцениваемые согласно гностической теории, удовлетворяют уравнению эквивалентности источников полей информации и термодинамической энтропии.

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