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GNOSTICAL APPROACH TO ROBUST CONTROL

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1 Summary

Data processing algorithms based on the gnostical theory of uncertain data possess high robustness with respect to both outlying data and changes of their statistical characteristics. There are several ways of substantial robustification of control systems by means of such algorithms. Effectiveness of this approach is demonstrated by examples.

2 Gnostical theory of uncertain data

Gnostical theory of uncertain data has been developed as an alternative of statistics for practical applications where nothing is known on a statistical model of the process, data are strongly disturbed, processes are non-stationary and there is lack of data to develop a statistical model. Algorithms based on the gnostical theory are inherently robust with respect to "outlying" data. They use no statistical assumptions related either to processes or to data. All necessary (gnostical) characteristics of the uncertainty are estimated directly from data. This is why a low sensitivity to changes of behaviour of disturbances is achieved by gnostical procedures. There are limits to the applicability of this theory connected with the validity of its axioms. However, these axioms express only basic algebraic requirements with respect to the nature of uncertainty.

3 Available gnostical algorithms

The following gnostical procedures have already been developed relevant for control applications:

- gnostical process monitors (treating time series, performing robust filtering of the process level, of its trend and of its acceleration, including robust diagnostics of the process);
- gnostical predictors for robust forecasting of disturbed processes;

- gnostical analysers of small data samples (for a detailed analysis of important data, such as estimating of probability of rare events, emergency limits for process control, random control of production quality, reliability studies, testing of homogeneity of objects and of their behaviour);
- gnostical identifiers of models (for robust identification of mathematical models of a process or object from disturbed observations).

These programs include some important auxiliary procedures such as estimation of scale parameters, data transformations, etc.

4 Robustification of control systems

There are several ways of robustifying control systems by means of gnostical procedures:

1) PID-control using gnostical filters to get robustly filtered proportional, integral and derivative signals;

2) robust filtering of the observed output level by the gnostical monitor preceeding the input of an optimum linear control system;

3) control using a robust gnostical predictor;

4) robust adaptive control based on the on-line identification of the system model by means of a gnostical identifier;

5) gnostical formulation of the optimum control problem with corresponding synthesis.

Methods 1) – 4) combine the linear methods of synthesis of regulators with robust treatment of signals. Method 5) promises maximum effect but it opens new theoretical problems because of nonlinearity: instead of the classical control error $e_c = z - z_0$ or $(z - z_0)/z_0$ (where z is the actual and z_0 the required output, $z, z_0 \in (0, \infty)$) a more complicated gnostical error function is used having the following form:

$$e_g = (q - 1/q)/(q + 1/q)$$

where $q = (z/z_0)^{2/s}$ and where s is a positive scale parameter. (Parameter s characterizes the intensity of random disturbances. It can be estimated from data).

Another important point is the quality of control. Within the framework of the gnostical theory there are functions of e_g available which evaluate the information loss and entropy gap caused by uncertainty of individual data. It could be interesting to optimize the control using these important criteria. It can be shown that it would result in high robustness.

5 Example 1

Examples demonstrate the effects of applying gnostics to control problems. Let us consider a continuous dynamic system of the type $1/(1+p)^3$ controlled by an LQ-optimal discrete-time self-tuned controller. In addition to a slowly changing disturbance, an uncorrelated random component exists due to which the system output is observed as noisy. We protect the input of the controller against the observation errors by means of a filter. Fig.1 shows the case of a strong observation noise modelled by the absolute value of disturbing signal of the Cauchy type, the filter being a "specialized" one prepared by the Bayesian statistical approach under an a priori assumption of Cauchy distribution. (We shall denote this type of filter as "Bayes/Cauchy"). The same system and disturbances filtered by the simplest gnostical monitor is in Fig.2. The actions of the controller are more modest here and the quality of the control is not worse. This takes place in spite of the fact, that the statistical filter makes use of the additional a priori information about the type of the distribution function which is not assumed by the gnostical filter. What happens after a change in the noise distribution function? The case of an absolute value of a weak uniformly distributed signal is in Fig.3 with the same gnostical filter. It is obvious from the Fig.4 that the control quality is not affected even by a substantial increase of variance of the uniformly distributed noise in the case of the gnostical filter. However, as seen in Fig.5, the system using the Bayes/Cauchy filter is unstable with this noise. The same happens when the noise is the absolute value of a strong Gaussian disturbance. The controller protected by the gnostical filter still works well (Fig.6) while it fails with the Bayes/Cauchy filter (Fig. 7) as well as without filter (Fig.8).

6 Example 2

This example is related to the application of an identifier within an adaptive control system. What is shown here is as follows: a gnostical identification procedure due to its robustness approaches the true parameters of the identified system in a much shorter time interval and with much smaller maximum errors than an unrobust (e.g. least squares) identification method. The signal under consideration is a series of real data representing rather complicated vibrations of a steam generator of an atomic power station. The problem is of the diagnostical type: to discover and classify changes of vibration modes as symptoms of dangerous states of the object. One of such test includes analysis of the difference between the actual and predicted values of the observed quantity. Necessary one–step–ahead predicted values are obtained by an AR–model of the 24–th order. The coefficients of this predictor are identified (estimated) by two methods:

a) ordinary least-squares procedure;

b) gnostical identification procedures according to the algorithm described in detail in [1].

To evaluate the quality of the identification process we introduce the notion of "noise amplification" A of a predictor having the form

$$y_{t+1} = \sum_{i=1}^{M} c_i y_{t-i+1}$$

(where y_j is the value observed in the j–th time interval, M is the order of the AR–model, c is the estimate of the i–th coefficient) by the equation

$$A = \sum_{i=1}^{M} c_i^2$$

It is interesting to analyse the time dependence of coefficient A resulting from applying both of the methods mentioned:

Tab.1 Noise amplification for two identification methods

Time	Number	Noise amplification A of the method	
(sec)	of data	least squares	gnostical
0.2	100	65000	4.63
0.4	200	7300	1.76
0.6	300	1160	1.51
0.8	400	300	1.48
1.0	500	114	1.42
1.2	600	49.9	1.46
1.4	700	9.42	1.42
1.6	800	5.75	1.44
1.8	900	2.69	1.42
2.0	1000	1.72	1.39

As this comparison shows, application of the robust gnostical identifier should result in better quality of the control.

References

[1] Kovanic P.: A new theoretical and algorithmical basis for estimation, identification and control, Automatica IFAC, 22, 6 (1986), 657-674

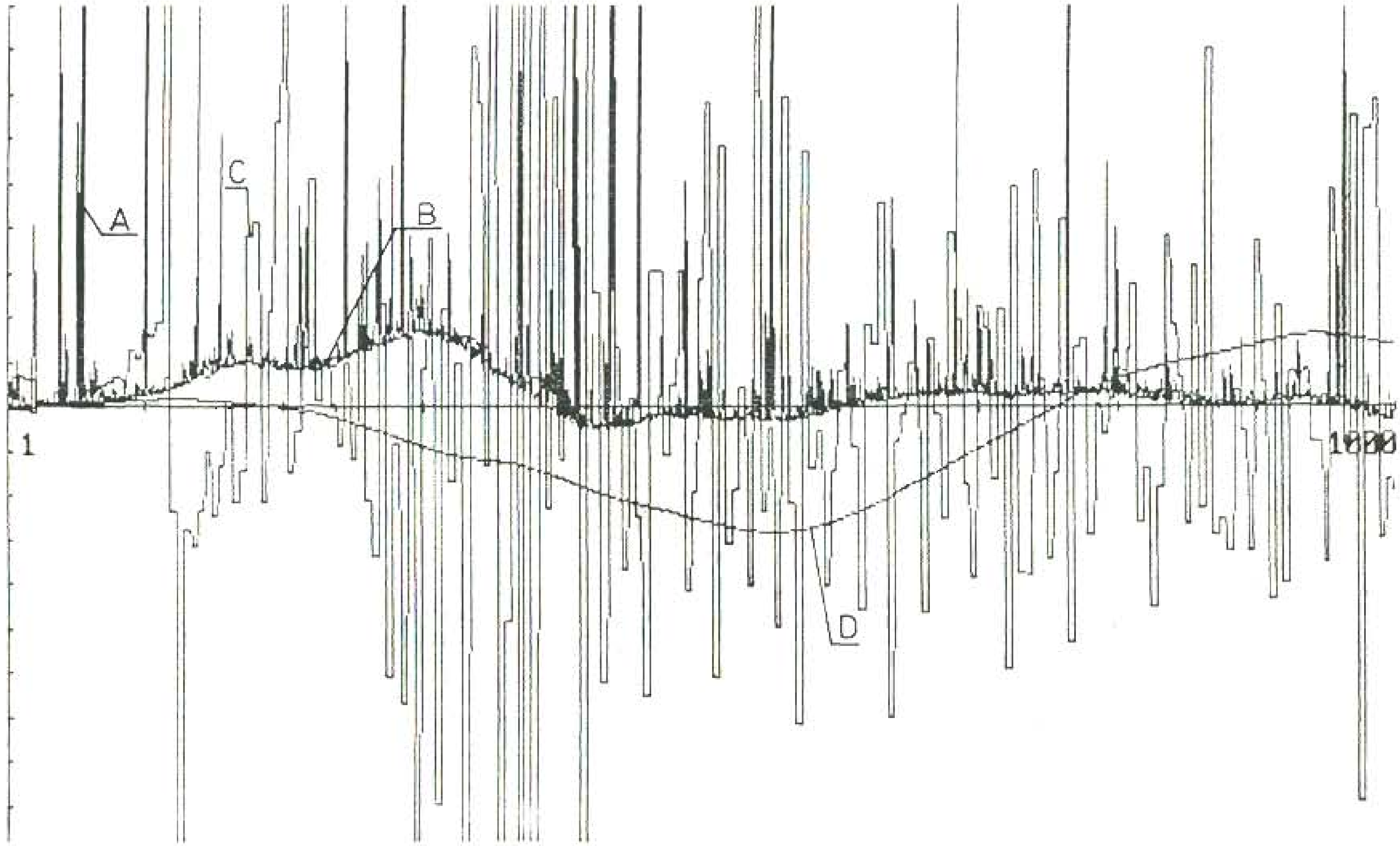


Fig.1: Automatic control under a random disturbance
A...observed (noisy) system output
B...filtered controller input
C...controller output
D...mean value of the disturbance
Distribution: ABS(Cauchy)
Filter: Bayes/Cauchy

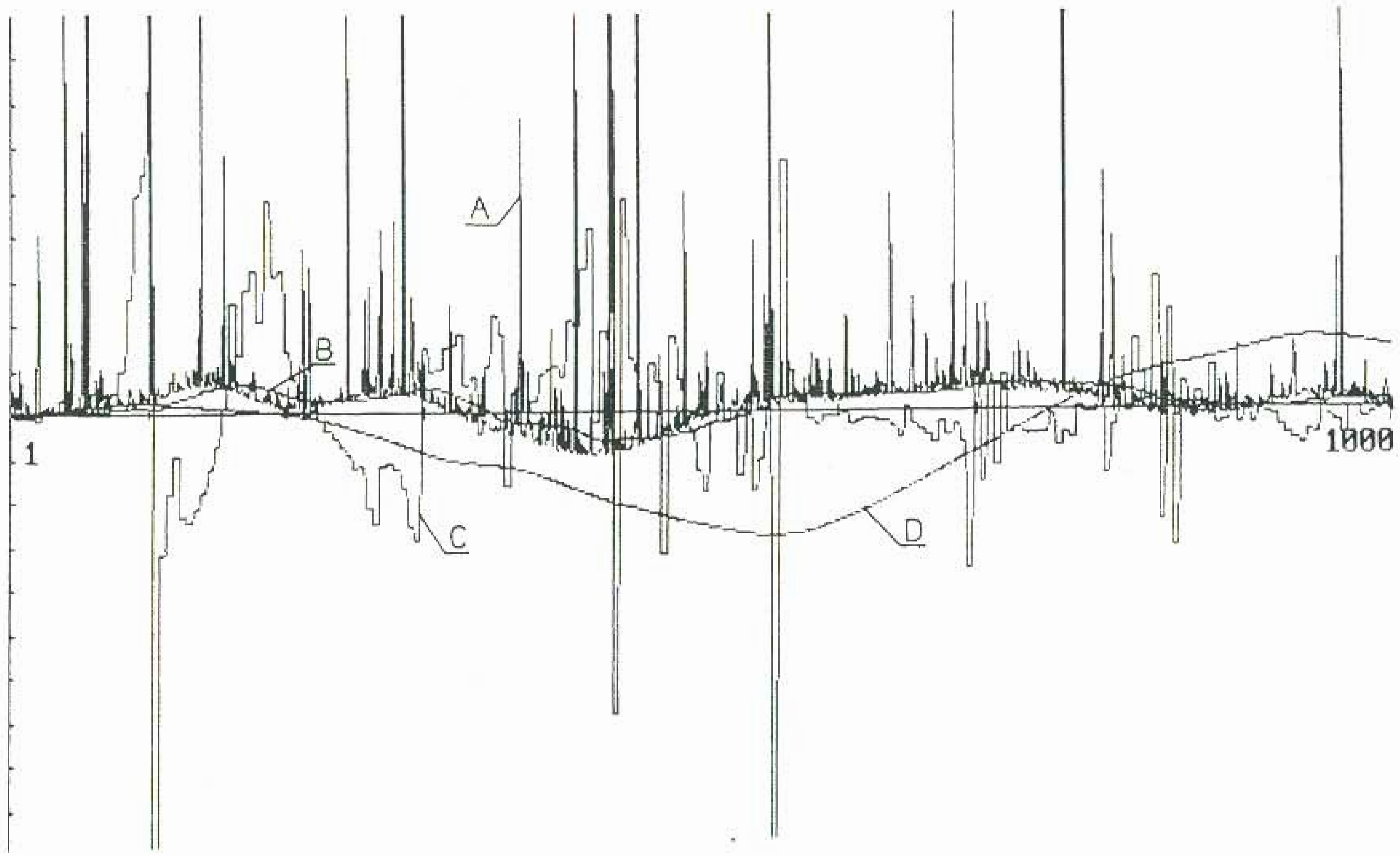


Fig.2: Automatic control under a random disturbance
A...observed (noisy) system output
B...filtered controller input
C...controller output
D...mean value of the disturbance
Distribution: ABS(Cauchy)
Filter: Gnostical

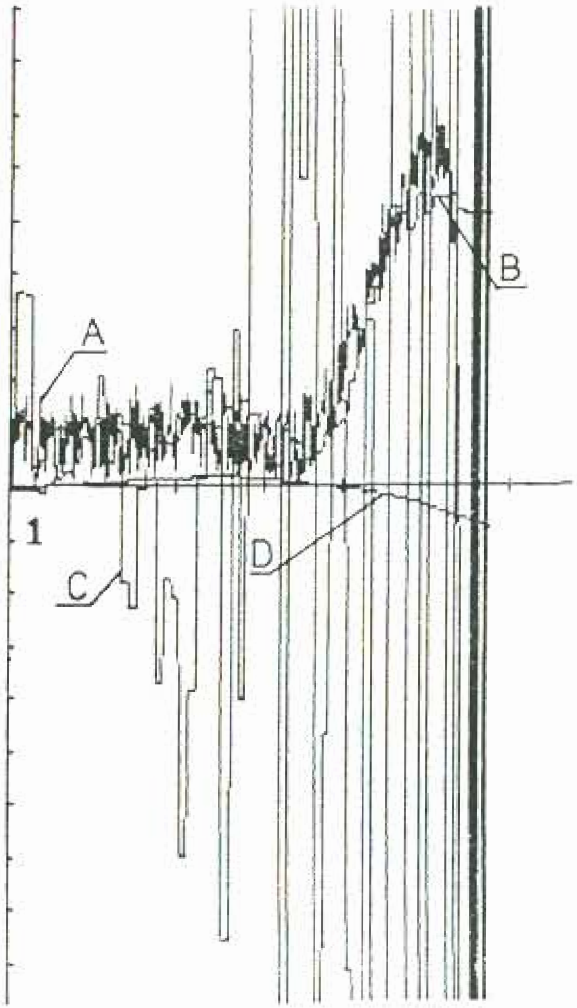
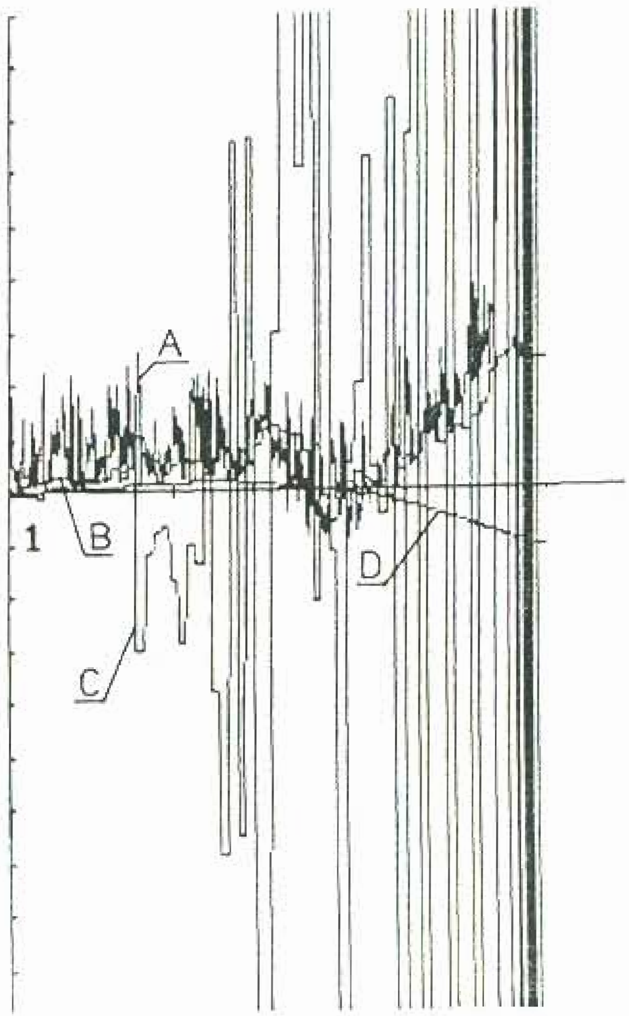


Fig.5:
Distribution: ABS(uniform)
Filter: Bayes/Cauchy



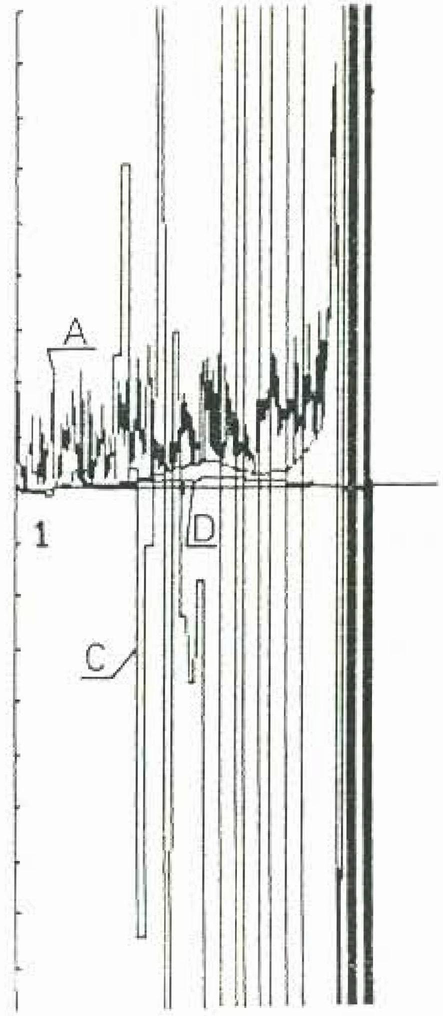


Fig.8:
Distribution: ABS(uniform)
Filter: None

Fig.7: Automatic control under a random disturbance
A...observed (noisy) system output
B...filtered controller input
C...controller output
D...mean value of the disturbance
Distribution: ABS(Gauss)
Filter: Bayes/Cauchy

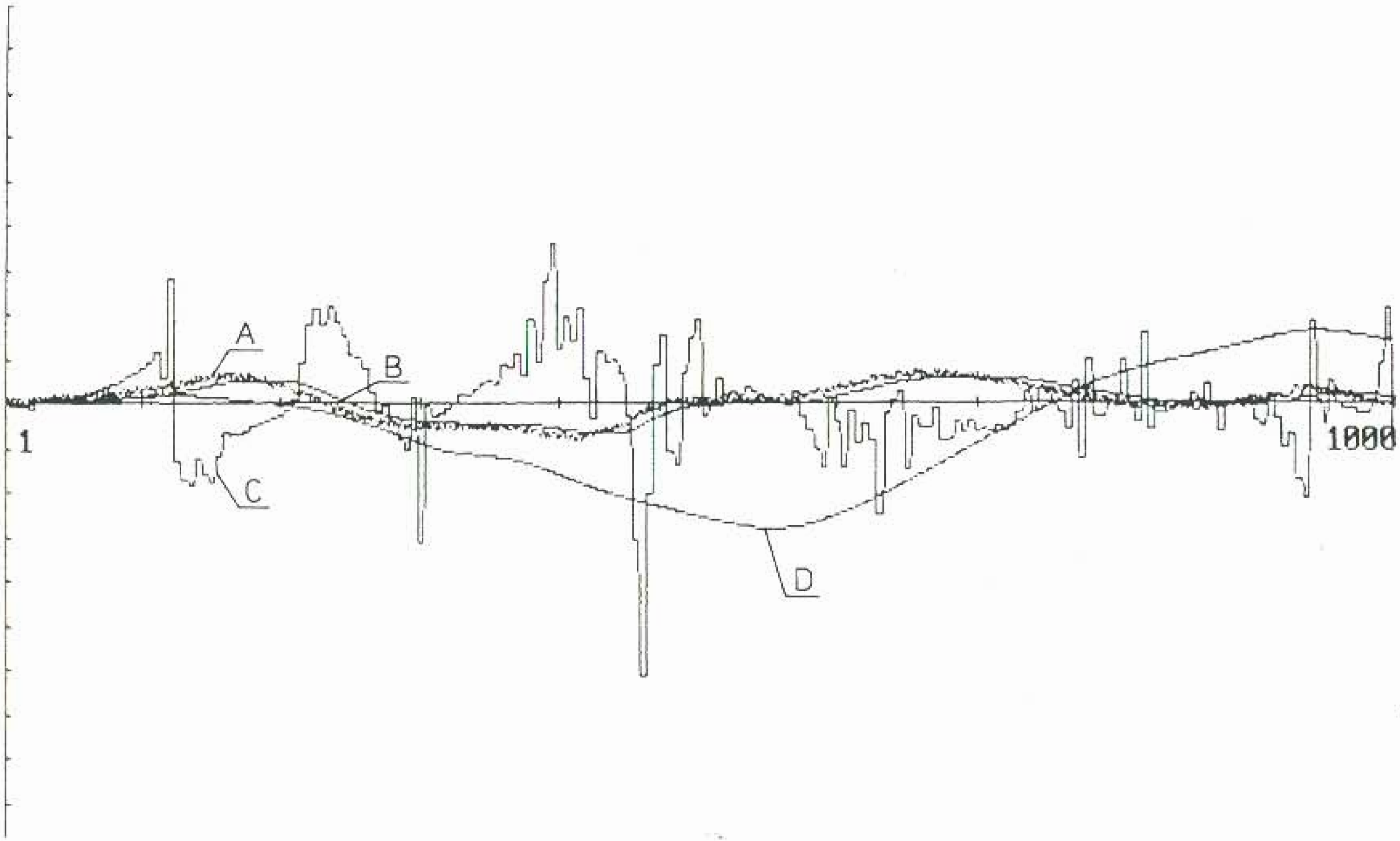


Fig.3: Automatic control under a random disturbance
A...observed (noisy) system output
B...filtered controller input
C...controller output
D...mean value of the disturbance
Distribution: ABS(uniform)
Filter: Gnostical

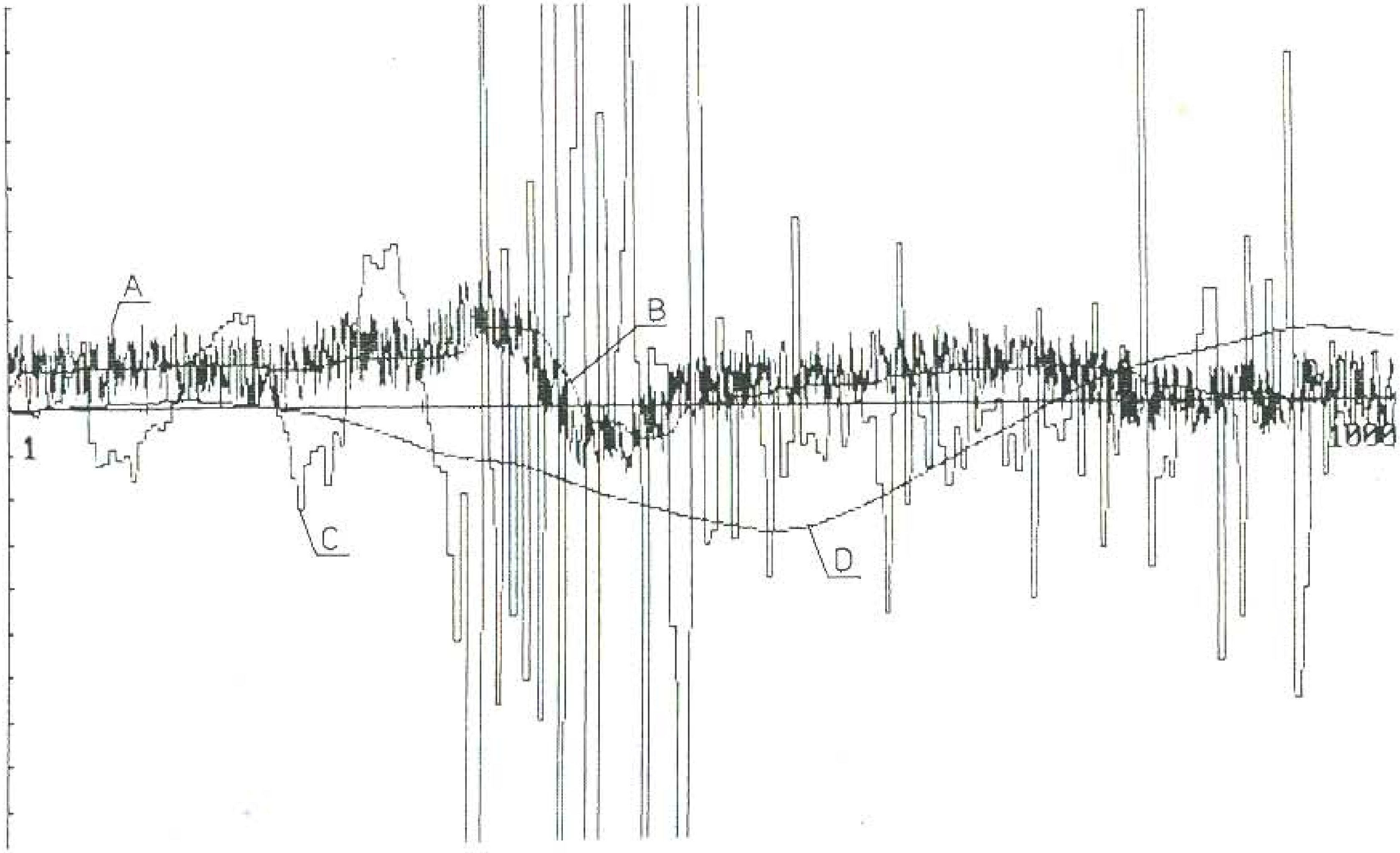


Fig.4: Automatic control under a random disturbance
A...observed (noisy) system output
B...filtered controller input
C...controller output
D...mean value of the disturbance
Distribution: ABS(uniform)
Filter: Gnostical

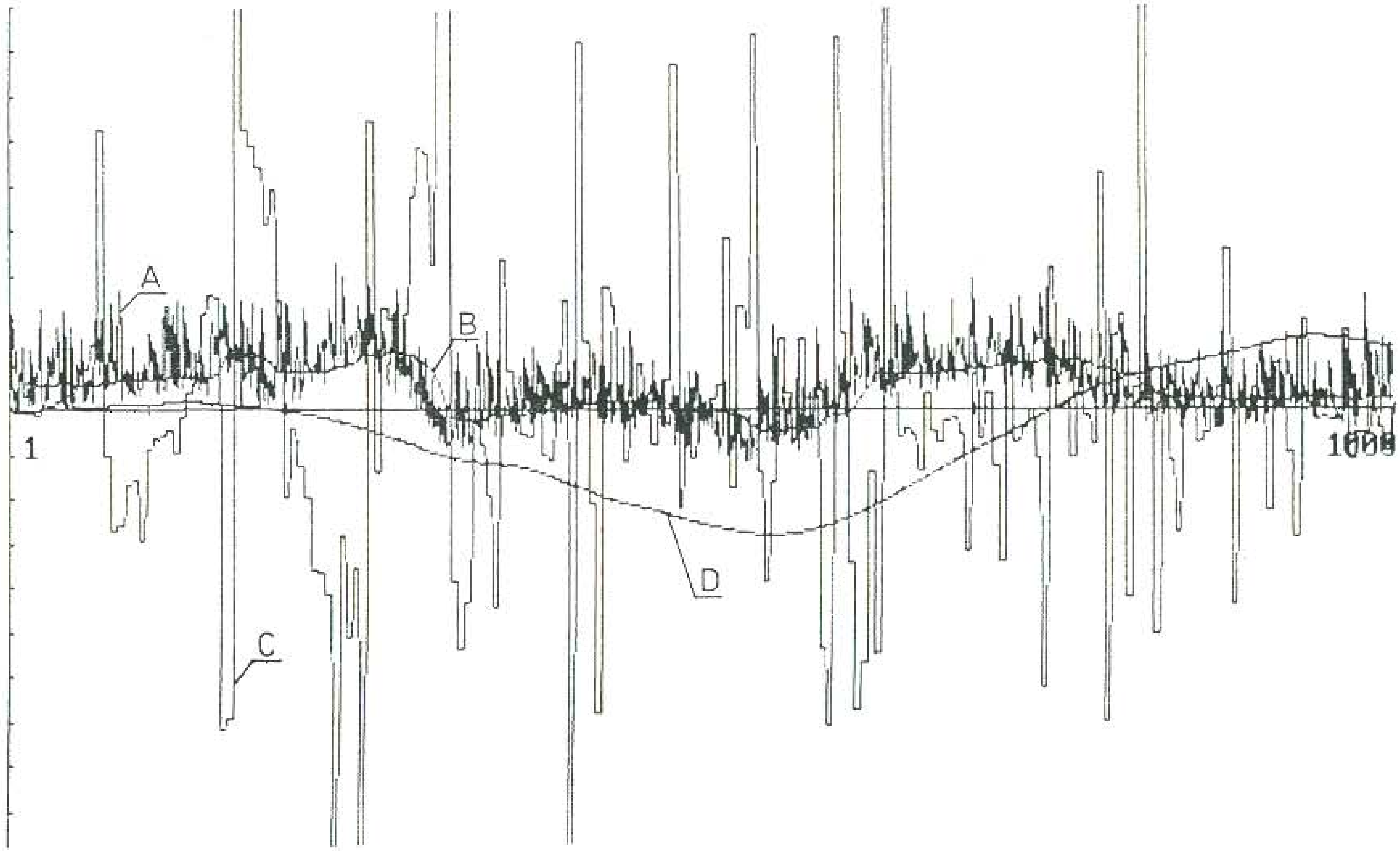


Fig.6: Automatic control under a random disturbance
A...observed (noisy) system output
B...filtered controller input
C...controller output
D...mean value of the disturbance
Distribution: ABS(Gauss)
Filter: Gnostical